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Control, Stability and Nonlinear Oscillations

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I. INTRODUCTION.

The principle areas of research carried out under this contract were: (1) Stability of dynamical systems, (2) Optimization theory (calculus of variations), (3) Stochastic variational problems, (4) Control theory and (5) Nonlinear Oscillations.

Section II of the report is a summary of the work completed and contains a brief statement of some of the principal results. References to complete reports on the research are given.

In almost all instances, complete reports on the research reported on below have been completed or are in preparation. With few exceptions, the results have been published in papers in scientific journals or have been accepted for publication or manuscripts are being prepared for submission for publication. A complete list of these papers and reports appears in the Bibliography at the end of the report.

II. SUMMARY OF WORK COMPLETED.

A. Stability of Dynamical Systems.

Liapunov's direct method is the only general method available for the investigation and study of the stability of dynamical systems that takes into account the nonlinearities of the system. Although there are a few general methods for the construction of Liapunov functions, this remains an open and important problem if the method is to be effectively employed by engineers and if we are to learn to use large-scale computers for carrying out stability analyses. Present methods require a large amount of ingenuity and, therefore, cannot be used by the average engineer interested in applications who is not a specialist in the field. Our major interest is in developing methods which are straight-forward, do not require ingenuity or advanced mathematical background, and are readily machine-computable.

The general problem is a very difficult one and little progress has been made, although the problem is now quite thoroughly understood. This work was done primarily by G. P. Szego, who is preparing a monograph on the subject.

On a more modest scale, considerable progress has been made by a generalization and improvement of methods first introduced by V. M. Popov ("Absolute stability of nonlinear systems of automatic control," Avt. i. Telemekh., 22 (1961), 961-979). It was shown by Kalman ("Lyapunov Functions for the Problem of Lur'e in Automatic Control," Proc. Nat. Acad. of Sci., 49, (1963), 201-205) that Popov's condition -- stated in the frequency domain -- assures the existence of Liapunov functions for systems which are linear except for a single nonlinear element. It was later shown also that the Popov condition is equivalent to the physical requirement of passivity. Hence, we now have a general method for the construction of Liapunov functions for passive systems containing a single nonlinear element. The importance of this work is emphasized by the fact that it is possible now to obtain -- in a mechanical, straightforward fashion -- about 75 per cent of all Liapunov functions which have been published so far (and which were constructed by ingenious tricks, rather than by general principles).

It appears that the above advances can be further generalized. While this work is now well under way by Kalman, final results have not yet been obtained.

A further, as yet unsolved, problem is the development of convenient methods for the machine computation of Liapunov functions. Whalen has studied the problem of replacing Popov's condition by an equivalent, time-domain condition which is more convenient for machine computation. This work is in progress. No significant results have been obtained to date.

V. Lakshmikantham has completed an extension of Liapunov's method to functional differential equations. In the case of ordinary differential equations, many problems concerning the behavior of solutions can be made to depend on scaler differential equations. Using a comparison principle of this type and the concept of a Liapunov function, he has extended many results to a general class of functional differential systems. A paper by Lakshmikantham entitled "Functional Differential Systems and Extension of Liapunov's Method" is to appear in the J. Math. Anal. Appl.

This same comparison principle mentioned above has been used by Lakshmikantham to investigate the problem of stability of solutions of parabolic differential equations. This problem has previously been investigated by Bellman, Prodi, Narasimhin and Mlak, among others. The advantage of Lakshmikantham's approach is that he obtains a large number of results in a unified way. For example, he obtains bounds on the solutions, uniqueness, stability and boundedness of solutions. His work illustrates also that using Liapunov-like vector functions can be quite useful. Examples are given to illustrate some of the results. A paper by Lakshmikantham entitled "Parabolic Differential Equations and Liapunov-like Functions" has been accepted for publication in the J. Math. Anal. Appl.

S. Lefschetz has been working for some time on a detailed account of the absolute stability of nonlinear control systems. He has incorporated in this study an account of the most recent work done by Popov, Yacubovich and Kalman. A manuscript has been prepared, and this will be the basis of a Monograph which will probably be published by McGraw-Hill. Copies of this manuscript are not yet available but could be supplied upon request.

Motivated by an improper application of Liapunov's methods in the study of the stability of an adaptive automatic control system, LaSalle developed two theorems which enable one to obtain qualitative information on the extent of the stability of this type of adaptive control system. In the design of control systems it is often the case that one is interested principally in reducing the error and may not be concerned with other parameters such as control parameters, as long as they remain bounded. Mathematically, this is the study of the stability of noncompact, that is nonbounded, invariant manifolds. A theorem for the study of this type of stability was developed by LaSalle. Simple examples were given illustrating how the theory can be applied. These results, along with a general study of eventual stability, were contained in a paper presented at the Second IFAC Congress at Basel, Switzerland, September 1963. A paper on this subject, entitled "Eventual Stability," will be published in the Proceedings of this Congress.

B. Optimization Theory (Calculus of Variations).

It is now well recognized that (deterministic) optimal control theory is essentially identical with the classical subject of the calculus of variations. For instance, the work of Pontryagin and his collaborators fits in quite well in the classical framework. However, the classical approach was primarily concerned with geometric problems while in control theory the emphasis is on problems from dynamics. We are now finding that the classical theory is incomplete in several areas where there are important problems from the point of view of control theory. Two such questions have been studied in some detail:

(a) Singular extremals. There are important cases where the Euler equations (or, equivalently, Pontryagin's Maximum Principle) do not provide any information concerning the nature of optimal paths. Such paths are usually called singular. Their study requires special methods. An important special case was studied by Wonham and Johnson, who obtained singular optimal paths for relay control systems. A paper on this subject entitled "Optimal Bang-Bang Control with Quadratic Performance Index" was presented at the 4th Joint Automatic Control Conference, 1963, University of Minnesota.

(b) The inverse problem of optimal control. Since the choice of the performance criterion to be minimized is always arbitrary and subjective, engineers have often complained that optimal control theory has only subjective value. In an effort to settle the controversy, the following question was investigated: Is a given control law optimal in any reasonable sense? Under highly restrictive but practically important conditions (linear systems with constant coefficients) this question can be completely resolved. These results were reported on in a paper by R. E. Kalman, entitled "When is a linear control system optimal?" presented at the 4th AACC Conference, 1963, University of Minnesota, and also appeared as a RIAS Technical Report No. 63-5, 1963.

It turns out that (i) optimality imposes severe restrictions on a control law and (ii) these restrictions require that the loop gain (or return difference) be kept as high as possible at all frequencies. The latter requirement was known since the work of Bode and is generally justified on the basis of reducing component variations. Thus, the engineer following Bode's theory will design a system which is optimal also in the dynamical sense, but without consciously realizing this! As a result of this work, all apparent contradictions between optimal control theory and engineering practice are now removed, and it is possible to place empirical rules of system design on a sound scientific basis.

Recent work along these lines shows that in the case of systems operating in a noisy environment current design practice is decidedly nonoptimal. This is an area where modern optimal control theory can be expected, in the not-too-distant future, to have an impact on practical system design.

R. A. Gambill completed a study of the existence theorems and optimal control theory from the point of view of the direct methods of calculus of variations. As is well known, and has already been indicated in this report, many mathematical formulations and problems in optimal control can be translated into a so-called Problem of Bolza in the calculus of variations.

In these problems one is concerned with the determination of a vector function, chosen from a prescribed class of functions which minimizes a given functional (usually an integral). The prescribed class usually consists of all those functions which satisfy given isoperimetric conditions, differential equations, inequalities, and end conditions. These conditions can also be thought of as functionals on the class of functions.

As far as the existence of a minimizing function is concerned, it is natural to examine the continuity (or semi-continuity) of the functionals involved, and the compactness of the class of functions. This may be done quite readily, using the notion of "generalized curves," a concept introduced by L. C. Young in 1937. Very briefly, a generalized curve is a vector function (defining a curve) together with a certain averaging process applied to the derivative of the function. This averaging process is carried out in such a way that every functional of the calculus of variations is continuous on the space of generalized curves, while at the same time preserving compactness of the space. Thus, theorems of existence of minimizing generalized curves for problems of Bolza are readily obtained. Certain additional restrictions on the functionals involved can then be made to assure that the minimizing generalized curve is also an ordinary curve. Slightly more general forms of several of the known existence theorems for optimal controls have been obtained by this method. Theorems of a similar nature, where the class of functions must also satisfy differential-difference equations and more general functional equations, are being studied by the same approach. A RIAS Technical Report TR 63-2 entitled "Generalized Curves and the Existence of Optimal Control" was prepared by R. A. Gambill. A paper based on this technical report has been submitted for publication.

The problem of minimizing the functional $\frac{1}{2}\int_0^T x_1^2 dt$ for the system $\ddot{x}_1 = u$, $|u| \leqq 1$, has been solved by a method based on the Hamilton-Jacobi equation. This research complements a paper of A. T. Fuller (Relay Control Systems Optimized for Various Performance Criteria, Proc. 1st International Congress of the IFAC, Butterworth, London, 1961, 510-519) where the same problem was studied by more special arguments. A paper on this subject entitled "Note on a Problem in Optimal Nonlinear Control" has appeared in the Journal of Electronics and Control, v. 15, No. 1 (1963) 59-62, by W. M. Wonham.

C. Stochastic Variational Problems.

Research was started under this contract on stochastic variational problems. Some preliminary results were reported in RIAS Technical Report 63-14 entitled "Stochastic Problems in Optimal Control," May 1963, by W. M. Wonham. Two types of Markovian control problems have been considered. In the first of these, the state of the system is assumed to be completely observable and sufficient conditions are given that a certain control law belonging to a class of admissible control laws be optimal; that is, yield the minimum expected value of a cost functional of integral type. The computation of an optimal control law is illustrated by an example. In the second problem, the admissible observations on the system are a function of time t, the state of the system and is subject to certain disturbances. The problem is to find a control law (functional) on the observations which minimizes the expected value of a cost functional, again of integral type. This is then a generalization of the first problem. The first step in the solution of this problem is to determine a finite number of statistics on the observations which are sufficient to specify completely the posterior probability distribution of the dynamical state at each instant. These statistics determine the controller's state of knowledge. If the evolution of these statistics with time can be described by a (stochastic) dynamical equation, and if for each admissible control law the joint process is Markovian, then the problem of determining an optimal control law can be solved using the methods for problem 1. This procedure was carried out in detail in TR 63-14 when the dynamical system is linear and the cost functional is quadratic.

D. Control Theory.

In the course of research on stability, optimization and stochastic variational problems, all of which have very definite and important applications, it was found necessary to make certain mathematical contributions to a more nebulous area, which for lack of a better word we shall call "general system theory." This work has had important theoretical implications and constitutes a major part of the research.

State variable vs. transfer functions. Although presently the state variable approach to system problems is exceedingly popular, many people have felt that the frequency response or transfer function approach has been unduly slighted. Indeed, there are good reasons for looking at problems in more than one way. To reconcile the two points of view, a long paper was written by Kalman ("Mathematical Description of Dynamical Systems," SIAM J. on Control, 1 (1963) No. 2) which appears to resolve this controversy once and for all. It was shown under the restrictions of complete controllability and complete observability -- which are easily satisfied in some cases but may be very restrictive in others -- that the two approaches are equivalent.

In the design of so-called "adaptive" systems, there are two natural criteria that any reasonable adaptive scheme should satisfy. The errors in identification should be bounded, or better yet, be asymptotically decreasing to zero as the amount of information increases. Furthermore, in cases where there are steady-state identification errors, the bound must be expressed in quantitative terms so that its effect may be properly evaluated. In addition, any scheme must have the ability to update continuously its control and identification based only upon the new data received. The computation for this updating should not increase with the total amount of data received, nor should it involve an excessive amount of work per cycle. The definition for "excessive" is, of course, relative to cost factors and to the present state of technology. B. H. Whalen and Yu-Chi Ho of Harvard University have examined an approach to the control of linear dynamic systems with unknown parameters from the point of view expressed above. Their primary objective was to indicate a possible method and to illustrate the type of difficulties that must be overcome and a quantitative and systematic study of "adaptive" control systems. This work by Ho and Whalen was reported on in a paper entitled "An Approach to the Identification and Control of Linear and Dynamic Systems with Unknown Parameters". This paper is to be published in IEEE Trans. on Automatic Control, Vol. AC-7, July, 1963.

In order to arrive at a better understanding of the fundamental aspects of control systems, Donald Bushaw has investigated a more general and more abstract description of a control mechanism. Under fairly liberal assumptions for any fixed control in a given class of admissible controls, the trajectories defined by the system of differential equations describing the control system define a dynamical system in the sense of Birkhoff, Markov, etc. on the event space. Thus, the whole class of admissible controls yields a class of dynamical systems on the event space. A short list of basic properties of such classes of dynamical systems has been formulated and provides axioms for what is called a "dynamical polysystem". Within this framework it is possible to formulate optimization problems and to obtain general forms of various basic results previously obtained by Roxin, Halkin, Bellman, and others. There is the hope that this theory will yield new insight into various aspects of the geometry of control theory. It seems possible that the new formulas may shed new light on questions of stability under continuously-acting disturbances and on the behavior of solutions of a system of differential equations when the solutions are not unique. Bushaw has prepared a RIAS Technical Report 63-10 entitled "Dynamical Polysystems and Optimization". A paper on this subject has also been accepted for publication in Contributions to Differential Equations.

E. Nonlinear Oscillations.

In the last few years, the theory of nonlinear oscillations for ordinary differential equations has been developing at a rapid rate. Many problems remain to be solved but some aspects of the theory are very well understood. Concurrently with the development of this theory there has been renewed interest in oscillatory phenomena for differential-difference equations. Jack K. Hale completed a study of how some of the known results for ordinary differential equations can be extended to differential-difference equations and showed very clearly the difficulties involved in such extensions. A paper on this research was presented at the EQUADIFF Conference in Prague, Czechoslovakia, September, 1962, and was entitled "Integral Manifolds and Nonlinear Oscillations".

In the study of the topological invariance associated with convex sets, it is important that large classes of sets which are topologically equivalent to convex sets be identified. This is particularly true in investigations which are concerned with the fixed-point property associated with continuous functions on convex sets. G. Stephen Jones has identified a class of topologically convex sets and uses this identification to obtain several interesting results in the theory of fixed-points. These results should have application in investigating periodic systems in Banach spaces. A report entitled "Topologically Convex Sets and Fixed-Point Theory" by G. Stephen Jones has appeared as RIAS Technical Report 63-8. A paper on this subject is being prepared by Jones for publication.

G. Stephen Jones has obtained results on the existence of manifolds of periodic solutions of a class of nonlinear autonomous functional differential equations. His technique involves defining a variable translator operator on a subclass of a space of possible initial functions and proving the existence of a fixed-point under this operator. The technique is nice in that it has the added feature of supplying a bound on the periods of these solutions. It's weakness lies in the fact that it may be difficult to define an appropriate translation operator. A RIAS Technical Report entitled "Periodic Motions in

Banach Space and Applications to Functional Differential Equations" has been prepared by G. Stephen Jones, TR 63-9. A revised version of this technical report is being prepared and will be submitted for publication.

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